

DISSIPATION IN INTERCLUSTER PLASMA

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ABSTRACT

We discuss dissipative processes in strongly gyrotropic, nearly collisionless plasma in clusters of galaxies (ICM). First, we point out that Braginsky (1965) theory, which assumes that collisions are more frequent than the system's dynamical time scale, is inapplicable to fast, sub-viscous ICM motion. Most importantly, the electron contribution to collisional magneto-viscosity dominates over that of ions for short-scale Alfvénic motions with wave length satisfying $l \leq \frac{\lambda}{\sqrt{\beta}} \left(\frac{m_e}{m_p} \right)^{1/4} \sim 1 \text{ kpc}$ (where λ is particle's mean free path, β is the plasma pressure parameter and $m_{e,p}$ are electron and proton masses). Thus, if a turbulent cascade develops in the ICM and propagates down to scales $\leq 1 \text{ kpc}$, it is damped collisionally not on ions, but on electrons.

Second, in high beta plasma of ICM, small variations of the magnetic field strength, of relative value $\sim 1/\beta$, lead to development of anisotropic pressure instabilities (firehose, mirror and cyclotron). Unstable wave modes may provide additional resonant scattering of particles, effectively keeping the plasma in a state of marginal stability. We show that in this case the dissipation rate of a laminar, subsonic, incompressible flows scales as inverse of plasma beta parameter. We discuss application to the problem of ICM heating.

Subject headings: galaxies: clusters: general

1. INTRODUCTION

One of the key problems in physics of intercluster medium (ICM) is the absence of strong cooling flows at the centers of galaxy clusters (see, *e.g.*, Peterson & Fabian 2006, for a review). It has been proposed that heating of ICM by Active Galactic Nuclei (AGNs) may be sufficient to offset the cooling (*e.g.* Begelman 2004). While the total energy budget of AGNs is, in principal, sufficient to offset the radiative cooling, details of how this is achieved are far from clear.

The observational confirmation of the AGN heating model comes from ubiquitous presence of AGN blown bubbles, identified by decreased X-ray emission in *Chandra* and XMM maps (McNamara 2000). These bubbles expand and rise in the cluster potentials transferring part of their energy to the internal energy of ICM. It has been suggested that this process can be very efficient, so that a large fraction of the power released by AGN ends up as internal energy of ICM.

The high efficiency of energy dissipation is far from obvious. What is required is a distributed increase of the entropy of the gas, not just of the internal or bulk energy (entropy floor problem Lloyd-Davies *et al.* 2000). The main problem is that these AGNs blown bubbles expand, typically, *subsonically*, as is indicated by the general absence of shock signatures ahead of the bubbles. In laminar flows at small Reynolds numbers, $Re \leq Re_{crit} \sim 10 - 100$, dissipation efficiency is $\propto 1/Re$. For $Re \sim 100$, such a low efficiency puts unreasonable demands on AGN luminosity.

2. COLLISIONAL DISSIPATION IN GYROTROPIC PLASMA: GYRORELAXATIONAL HEATING

Ion Larmor radii in ICM, $r_L \sim 10^8 \text{ cm}$, is some fifteen orders of magnitude smaller than the system size, $L \sim \text{hundreds of kpc}$, and Coulomb mean free path,

$\lambda \sim 10 - 30 \text{ kpc}$, for a typical density $n \sim 10^{-3} \text{ cc}$, magnetic fields $\sim 1 - 10 \mu\text{G}$ and temperatures in the keV range (*e.g.* Carilli & Taylor 2002). Thus ICM is weakly collisional, $r_L \ll \lambda$. In addition, it is weakly magnetized, in a sense that magnetic fields energy is smaller than plasma pressure, $\beta = 8\pi P/B^2 \geq 1$. Below we will refer to this regime as a strongly gyrotropic plasma.

Dissipation in a strongly gyrotropic plasma proceeds in a qualitatively different way from the isotropic case, as is exemplified by so called gyrorelaxational heating. If in an initially pressure-isotropic plasma the absolute value of magnetic field oscillates with a frequency ω and relative amplitude $\delta = \delta B/B_0$, then the dissipation rate α (so that energy of a particle \mathcal{E} changes according to $d\mathcal{E}/dt = \alpha\mathcal{E}$) in a cycle is (eg Borovsky 1986)

$$\alpha \approx \frac{\omega^2 \nu_c}{\frac{9}{4} \nu_c^2 + \omega^2} \frac{\delta^2}{6} \quad (1)$$

where ν_c is collision frequency. Dissipation of energy occurs both due to electron and ion collisions, so that $\alpha = \alpha_e + \alpha_i$, calculated with corresponding collision frequencies ν_e and ν_i . In the high collision frequency regime, $\nu_c \gg \omega$, Eq. (1) approximates Braginsky's result, $\alpha \propto 1/\nu_c$ Braginsky (1965). Since ions have smaller collision frequency, dissipation in this limit is dominated by ions. On the other hand, *for rare collisions*, $\nu_c \ll \omega$, *dissipation rate is proportional to collision frequency*, $\alpha \propto \nu_c$, *and is thus dominated by electrons* for $t_e \omega > \frac{3}{2} (m_e/m_i)^{1/4} \approx 0.2$

Consider sub-viscous turbulent motion of ICM occurring on scale l smaller than mean free path λ and mediated by Alfvén waves, so that a typical wave frequency is $\omega = V_A/l \sim c_s/(\sqrt{\beta}l)$. Then for waves satisfying $t_e \omega > \frac{3}{2} (m_e/m_i)^{1/4}$, or for

$$l \leq \frac{\lambda}{\sqrt{\beta}} \left(\frac{m_e}{m_p} \right)^{1/4} \sim 1 \text{ kpc}. \quad (2)$$

electron viscosity dominates over ion viscosity. For numerical estimates we assumed $T = 10^8$ K, $n = 10^{-3}$ cc, $B = 5 \mu$ G, so that $\beta \sim 10$ and mean free path $\lambda = 23$ kpc.

Thus, if a turbulent cascade develops in the ICM and propagates down to scales ≤ 1 kpc, it is damped collisionally not on ions, but on electrons. Thus, *Braginsky (1965) theory, which assumes frequent collisions, $t_{\text{coll}}\omega \gg 1$, is inapplicable to fast, sub-viscous ICM motion.*

3. HEATING IN A BOUND ANISOTROPY MODEL

Besides binary collisions, plasma can be heated through development of electromagnetic turbulence which resonantly scatters particles and, thus, provides an additional dissipation mechanism. In this Section we describe such mechanism of dissipation through development anisotropic plasma instabilities.

3.1. Viscosity due to binary collisions in a gyrotropic plasma

When Coulomb collision frequency ν_c is much smaller than cyclotron frequency, $\omega_B/\nu_c \gg 1$, plasma viscosity is strongly anisotropic, determined by seven coefficients (Braginsky 1965). In the limit $\omega_B \rightarrow \infty$ and slow changes of magnetic field, $\omega \ll \nu_c$, the only remaining coefficient is η_0 , responsible for the viscosity along the field lines. In this case the viscous stress tensor becomes (Landau & Lifshits 1982)

$$\sigma_{ij} = \eta_0 (3b_i b_j - \delta_{ij}) \left(b_l b_k \partial_l v_k - \frac{1}{3} \text{div} \mathbf{v} \right) \quad (3)$$

where b_i is a unit vector along the local magnetic field, $\eta_0 = p/\nu_c$, $p = (P_{\parallel} + 2P_{\perp})/3$ is total pressure. Below we concentrate on the incompressible limit, $\text{div} \mathbf{v} = 0$, which eliminates reversible compressional heating. For incompressible plasma without conductivity, using Eq. (3), the volumetric dissipation and entropy generation rates due to viscosity are (Landau & Lifshitz 1975)

$$\rho \frac{dE}{dt} = \rho T \frac{dS}{dt} = \sigma_{ij} \partial_i v_j = 3\eta_0 (\mathbf{b} \cdot (\mathbf{b} \nabla) \mathbf{v})^2 \quad (4)$$

Using induction equation, $d\mathbf{B}/dt = (\mathbf{B} \nabla) \mathbf{v}$, the entropy generation rate can be related to the rate of change of magnetic field (Schekochihin & Cowley 2006):

$$\rho \frac{dE}{dt} = 3\eta_0 \left(\frac{1}{B} \frac{dB}{dt} \right)^2 \quad (5)$$

Dissipated power of a gyrotropic fluid is solely due to changing magnetic field, which is very different from the isotropic case. This result can also be verified if we note that in a gyrotropic plasma the entropy is $S \propto (1/2) \ln P_{\perp} P_{\parallel}^2$ (assuming constant density). The entropy production is then

$$\frac{dS}{dt} = \frac{1}{3} \frac{(P_{\perp} - P_{\parallel})^2}{P_{\perp} P_{\parallel}} \nu \quad (6)$$

For binary collisions using $P_{\perp} - P_{\parallel} = 3\eta_0 d_t \ln B$ (Eq. (12)), this gives

$$\frac{dS}{dt} = 3(d_t \ln B)^2 \eta_0^2 \frac{\nu}{P_{\perp} P_{\parallel}} \approx 3 \frac{(d_t \ln B)^2}{\nu} \quad (7)$$

consistent with (5).

The differences between the dissipation rates calculated using isotropic and anisotropic viscosities can be dramatic. For example, for spherical expansion of a bubble into incompressible fluid, in absence of magnetic field, the dissipated power is zero (flow field is irrotational). Introduction of a weak (in a sense that $\beta \gg 1$) magnetic field changes this picture completely. In a kinematic approximation (neglecting its dynamical effects, so that field lines are just advected with the flow satisfying frozen-in condition) expansion of a bubble into a constant magnetic field creates magnetic fields

$$B_{\theta} = \frac{\sin \theta}{(1 - \xi^{-3})^{1/3}} B_0, \quad B_r = -\cos \theta (1 - \xi^{-3})^{1/3} B_0 \quad (8)$$

where $\xi = r/R(t) > 1$ and B_{θ} and B_r are component of magnetic field in a spherical system of coordinates aligned with the initial direction of magnetic field. Though tangential component of magnetic field diverges on the contact $\xi = 1$ (magnetic draping effect), the increase in B-field energy over the initial homogeneous field is finite, $(1/9)B_0^2 R^3$ and the total heating rate is $d_t E = 3\eta_0 R^3 \int d^3 \xi (d_t \ln B)^2 = 9.54 \eta_0 R^3 (d \ln R/dt)^2$.

This example illustrates an important point: even a weak magnetic field may considerably affect plasma dissipative properties. Inverse situation, when a dissipative flow with isotropic viscosity becomes non-dissipative in the strongly gyrotropic limit, is also possible. The example is a longitudinal shear, when magnetic field is directed along velocity. In the absence of cross-field viscosity there is no dissipation.

3.2. Anisotropic pressure instabilities

In collisionless plasma, particles in magnetic fields tend to conserve their adiabatic invariants (Chew *et al.* 1956). In case of rare collisions the equations describing evolution of pressures becomes (eg Hollweg 1985)

$$\begin{aligned} \frac{d \ln P_{\perp}/B}{dt} &= \frac{\nu}{3} \frac{P_{\parallel} - P_{\perp}}{P_{\perp}} \\ \frac{d \ln P_{\parallel} B^2}{dt} &= -\frac{2\nu}{3} \frac{P_{\parallel} - P_{\perp}}{P_{\parallel}} \end{aligned} \quad (9)$$

where P_{\perp} and P_{\parallel} are pressure across and along magnetic field.

In a $\beta \gg 1$ plasma, the development of pressure anisotropy may lead to firehose, mirror and ion cyclotron instabilities when the following conditions are satisfied

$$\beta_{\parallel} - \beta_{\perp} > 2, \text{ firehose, } \frac{\beta_{\perp}}{\beta_{\parallel}} > 1 + \frac{1}{\beta_{\perp}} \text{ mirror} \quad (10)$$

and $\beta_{\perp}/\beta_{\parallel} > 1 + k/\beta_{\parallel}^m$ cyclotron, where $\beta_{\parallel} = 8\pi P_{\parallel}/B^2$, $\beta_{\perp} = 8\pi P_{\perp}/B^2$, $0.35 \leq k \leq 0.65$ and $0.4 \leq m \leq 0.42$ Gary *et al.* (1994). Cyclotron instability has growth rate larger than the mirror instability for $\beta \leq 6$ and $p_{\perp} > p_{\parallel}$. If initially plasma pressure is isotropic, firehose and mirror instability occur when $\delta B/B = -2/(3\beta_0)$ (firehose), $\delta B/B = +1/(3\beta_0)$ (mirror) and similar expression for ion cyclotron instability (Gary *et al.* 1994); for clarity we do not consider it here.

The instabilities' increment is maximal at the cyclotron frequency, which is very fast compare to any dynamical

time. Further change of magnetic field, beyond the limits (3.2), will be accompanied by development of instabilities which will lead to increased scattering rate, either due to quasi-linear diffusion or a fully developed turbulence. As a result, the system dissipates quickly any free energy in excess of instability threshold and relaxes to the marginally stable state. We expect that the system remains at threshold of instability.

3.2.1. Binary collisions in sub-critical regime

Binary collisions decrease a level of anisotropy and may stabilize plasma. Redefining pressures P_\perp and P_\parallel in terms of total pressure p (a trace of the pressure tensor) and pressure disbalance $(P_\perp - P_\parallel)/p = \Delta$, $P_\parallel = p - \frac{2}{3}\Delta p$, $P_\perp = p + \frac{1}{3}\Delta p$, we find

$$2p\Delta \frac{dB}{dt} = 3B \frac{dp}{dt} \quad (11)$$

$$\frac{d\Delta}{dt} + \nu\Delta - \frac{9 - 3\Delta - 2\Delta^2}{3} \frac{d \ln B}{dt} = 0$$

In a $\beta \gg 1$ plasma at the moment of instability Δ is small, $|\Delta| \ll 1$. For slow changes $d/dt \ll \nu$ this gives

$$\nu\Delta = 3 \frac{d \ln B}{dt} \quad (12)$$

This implies that for development of instabilities the dynamical time $t_{dyn} \sim 1/dt \ln B$ should be relatively short, $t_{dyn}\nu_c \leq \beta$. This condition is satisfied by most scales of interest in ICM plasma.

3.3. Dissipation at marginal stability

As we argued in the previous section, changing magnetic field will lead to development of instabilities that will keep the plasma anisotropy at the critical values given by Eqs. (10). Eqs. (9) and the condition (10) may be regarded as defining an effective scattering rates

$$\nu_{eff, firehose} = \frac{3}{2}\beta d_t \ln B, \quad \nu_{eff, mirror} = 3\beta d_t \ln B \quad (13)$$

At a critically balanced case, the entropy generation rate Eq. (6)

$$d_t S \approx \frac{2}{\beta} d_t \ln B \times \left(\frac{1}{\frac{1}{2}} \right) \quad (14)$$

for the firehose and mirror regimes.

We have arrive at an important result related to efficiency of dissipation: *in a gyrotropic plasma efficiency of dissipation is determined not by Reynolds number, but by the plasma beta parameter*. Typical dissipation time scale is β times dynamical time, not Re times dynamical time.

The role of effective collisions in energy dissipation in a marginally stable regime is, in some sense, opposite to the role of binary collisions in a sub-critical regime. The entropy production rate and corresponding volumetric dissipated power, Eq. (6), are proportional to pressure anisotropy and collision frequency, $\propto \nu(\Delta P)^2$, where ΔP is the difference in parallel and transverse pressures. If the pressure disbalance is due to binary collisions, then $\Delta P \propto 1/\nu$ so that the dissipation rate is $\propto 1/\nu$ (Braginsky 1965). Thus, before the instabilities are reached, increasing collision rate leads to decreasing dissipation. On the other hand, for marginally stable case $\Delta P \sim \text{constant}$, so that dissipated power is *proportional* to the effective collision rate, Eq. (7).

3.4. Damping of waves at marginal stability

For Alfvén waves, perturbations of magnetic field are orthogonal to the initial magnetic field, so that variations of the absolute value of the field are second order in amplitude. For large enough amplitude, satisfying condition $(\delta B/B_0)^2 \equiv \delta^2 > 1/\beta$, this creates conditions favorable for mirror instability. The entropy production rate over the period is

$$\frac{dS}{dt} = \frac{4}{\beta} \frac{\delta^2}{1 + \delta^2} \omega \left(\frac{2 \arccos \frac{1}{\beta \delta^2}}{\pi} \right) \quad (15)$$

where the term in parenthesis takes into account phases when the amplitude of fluctuations satisfies the mirror instability criterion. With Braginsky viscosity, the collisional damping of Alfvén waves is a non-linear effect as well, but it has a much steeper dependence of wave amplitude and frequency. From (5) we find

$$\frac{dS}{dt} = \frac{3\delta^4 \omega^2}{(1 + \delta^2)^2 \nu_c} \quad (16)$$

For comparison, in isotropic MHD Alfvén waves are damped at a rate (Landau & Lifshitz (1982)) $dS/dt = \delta^2 \omega^2 / \nu_c$.

4. DISCUSSION

Our approach follows a long established procedure of marginal stability Kennel & Petschek (1966); Manheimer & Boris (1977); Gary *et al.* (1994); Denton *et al.* (1994), when the instability threshold becomes the limiting value of anisotropy. In particular, Quest & Shapiro (1996); Gary *et al.* (1998) applied a bounded anisotropy model to the measurements of parallel and perpendicular temperatures in the solar bow shock region near the Earth magnetosphere. It was found that an initial rapid growth of unstable waves indeed brings the system back to approximate marginal stability.

What is the relation of the marginal stability condition and the conventional quasilinear and turbulence theories? According to Manheimer & Boris (1977), both predict some level of turbulent fluctuations. Marginal stability approach is applicable if the level of those fluctuations is smaller than the one calculated from non-linear theory. This, typically, happens when the driver of the instability (in our case a large scale motion of ICM plasma) is not strong. Assessing whether this is satisfied in case of ICM plasma requires full scale calculations of non-linear turbulence levels, a prohibitively complicated task given the uncertainties in both plasma microphysics and details of ICM plasma motions.

The most important effect that was not taken into account in the present work is thermal conduction. The double-adiabatic equations are valid only when heat flux along magnetic field lines can be neglected. This is the main reason why the theory may fail (*e.g.*, the notorious results of Kulsrud *et al.* 1965). Neglect of heat flux requires that phase velocity of the perturbations be much larger than speed of heat carriers, electrons: $(\frac{\omega}{k})^2 \sim V^2 \gg v_{T,e}^2$. This condition may be broken in ICM, especially outside of cluster cores. On the other hand, enhanced scattering rate suppresses conductivity (Levinson & Eichler 1992). The conduction coefficient is

$\kappa \sim n_e v_{T,e}^2 / \nu_{eff}$ (assuming that saturate conductivity regime (Cowie & McKee 1977) is not reached). The effective scattering frequency due to development of electromagnetic instabilities Eq. (13) may be higher than binary collision rate, so that the conduction coefficient will be smaller, $\kappa \sim n_e v_{T,e}^2 L / (\beta V)$. Increased scattering will also inhibit the onset of saturated regime.

There is a number of challenges that heating models should overcome. Primarily, the heating must be both widely distributed and gentle. It is hardly achievable with shocks, which provide very concentrated heating at the shock location, deposit most of the energy in the core and generally contradict the observational absence of shock signatures. This, combined with low heat conductivity in the cores, leads to plasma overheating and creation of inverted entropy gradients, contrary to observations (*e.g.* Voit & Bryan 2001).

The heating in the bounded anisotropy model may be distributed. Consider a cluster with a typical density profile $\rho \propto 1/r$. Then if bremsstrahlung dominates over line emission, the cooling rate is $\propto r^{-2}$ (for nearly constant temperature in the cores). Since an energy flux

from central source scales as $\propto r^{-2}$ as well, this implies that a heating rate should be independent of a radius, and thus independent of the local plasma properties. Collisional dissipation clearly cannot produce this. On the other hand, if β is nearly constant, the heating rate will be nearly independent of radius. Thus, at least in principle, heating and cooling can be balanced in the bound anisotropy model.

One of the main drawbacks of many simulations of ICM is that they use isotropic Spitzer viscosity. Examples in §3 show that this can produce (at least locally) drastically incorrect results, which may either overestimate or underestimate the real collisional magnetoviscosity (we are not aware of any ICM-related simulations with anisotropic viscosity (see, though, Sharma *et al.* 2006)). As for the value of the coefficient of viscosity, we argued that for binary collision it generally depends on electron and ion temperatures and dynamical times scales, while in case of marginal stability it is actually unrelated to the Spitzer value. Parametrization with respect to Spitzer may be useful, but we should not put too much physical emphasis on it.

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